11.4 LINES + PLANES IN 3D

IN 2D, YOU NEED THE SLOPE + A POINT TO IDENTIFY A LINE
(OF THE LINE, ON THE LINE)

$$m = \frac{\Delta y}{\Delta x}$$

IN 3D, YOU NEED A VECTOR THAT IS PARALLEL TO THE LINE
+ A POINT ON THE LINE
TO IDENTIFY A LINE

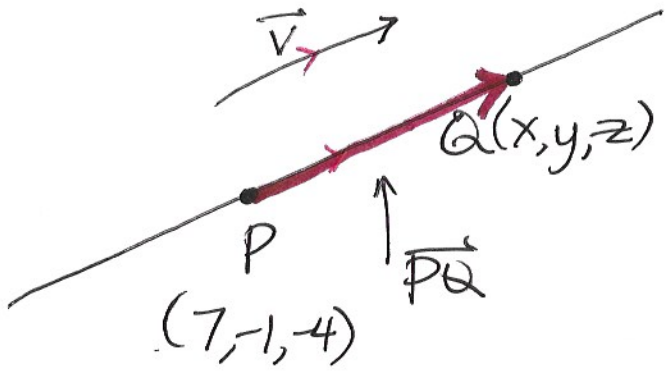
A/K/A DIRECTION VECTOR

eg. FIND EQUATIONS FOR THE LINE PARALLEL TO $\vec{v} = \langle -2, -5, 3 \rangle$ PASSING THROUGH $P(7, -1, -4)$.

COMPONENTS OF VECTOR

$$\begin{aligned} x &= 7 - 2t \\ y &= -1 - 5t \\ z &= -4 + 3t \end{aligned}$$

POINT ON LINE



LET $Q(x, y, z)$ BE ON THE LINE

$\overrightarrow{PQ} \parallel \vec{v}$ — CRITICAL CONCEPT → $\overrightarrow{PQ} = k\vec{v}$

$\langle x-7, y-(-1), z-(-4) \rangle = k\langle -2, -5, 3 \rangle$

$\langle x-7, y+1, z+4 \rangle = \langle -2k, -5k, 3k \rangle$

$$\begin{aligned} x - 7 &= -2k & x &= 7 - 2k \\ y + 1 &= -5k \longrightarrow & y &= -1 - 5k \\ z + 4 &= 3k & z &= -4 + 3k \end{aligned}$$

PARAMETRIC FORM OF EQ'N OF A LINE

THE PARAMETRIC (FORM OF THE) EQUATIONS OF THE LINE WITH DIRECTION VECTOR $\vec{d} = \langle a, b, c \rangle$ PASSING THROUGH POINT (x_0, y_0, z_0) IS

$$x = x_0 + at$$
$$y = y_0 + bt$$
$$z = z_0 + ct$$

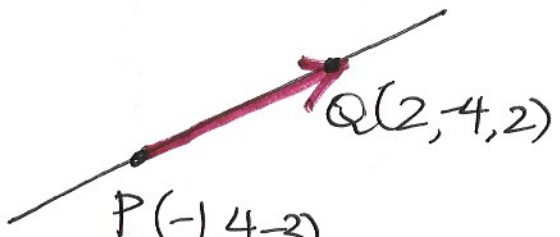
eg. FIND THE PARAMETRIC EQUATIONS OF THE LINE THROUGH $(2, 0, -5)$ AND PARALLEL TO $\vec{v} = \langle 0, 3, 7 \rangle$

$$x = 2 + 0t \qquad x = 2$$
$$y = 0 + 3t \longrightarrow y = 3t$$
$$z = -5 + 7t \qquad z = -5 + 7t$$

eg. FIND THE PARAMETRIC ... THROUGH $(-1, 4, -3)$ AND $(2, -4, 2)$

$$\vec{d} = \overrightarrow{PQ} = \langle 2-1, -4-4, 2-(-3) \rangle = \langle 3, -8, 5 \rangle$$

OR $k\langle 3, -8, 5 \rangle$

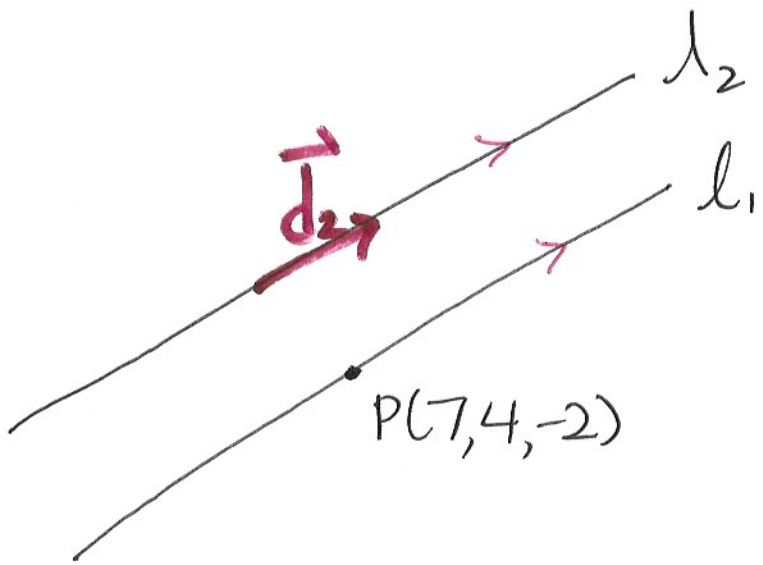


$$x = 2 + 3t$$
$$y = -4 - 8t \quad \leftarrow \text{USED } Q \text{ AND } \overrightarrow{PQ}$$
$$z = 2 + 5t$$

(COULD HAVE USED P AND $\overrightarrow{PQ}$, OR Q AND $\overrightarrow{QP}$

eg. FIND PARAMETRIC EQUATIONS FOR THE LINE THROUGH $P(7,4,-2)$ $\leftarrow l_1$ AND PARALLEL TO

$$\left.\begin{aligned} x &= 2t-5 \\ y &= 7 \\ z &= -8t \end{aligned}\right\} l_2$$



$\vec{d}_1$ = DIR VEC OF l_1

LET $\vec{d}_1 = \vec{d}_2 = \langle 2,0,-8 \rangle$

OR $\vec{d}_1 = \frac{1}{2}\langle 2,0,-8 \rangle$

$= \langle 1,0,-4 \rangle$

$$\begin{aligned} x &= 7+1t \\ y &= 4+0t \\ z &= -2-4t \end{aligned} \quad \rightarrow \quad \begin{aligned} x &= 7+t \\ y &= 4 \\ z &= -2-4t \end{aligned}$$

ELIMINATING THE PARAMETER

POINT $(7,-1,-4)$

$\vec{d} = \langle -2,-5,3 \rangle$

$$\begin{cases} x = 7-2t \longrightarrow x-7 = -2t \longrightarrow t = \dfrac{x-7}{-2} \\ y = -1-5t \longrightarrow y+1 = -5t \longrightarrow t = \dfrac{y+1}{-5} \\ z = -4+3t \longrightarrow z+4 = 3t \longrightarrow t = \dfrac{z+4}{3} \end{cases}$$

$$\frac{x-7}{-2} = \frac{y+1}{-5} = \frac{z+4}{3}$$

SYMMETRIC FORM OF EQ'N OF A LINE

THE SYMMETRIC FORM OF THE EQUATIONS OF THE LINE WITH DIRECTION VECTOR $\vec{d} = \langle a,b,c \rangle$ PASSING THROUGH POINT (x_0, y_0, z_0) IS

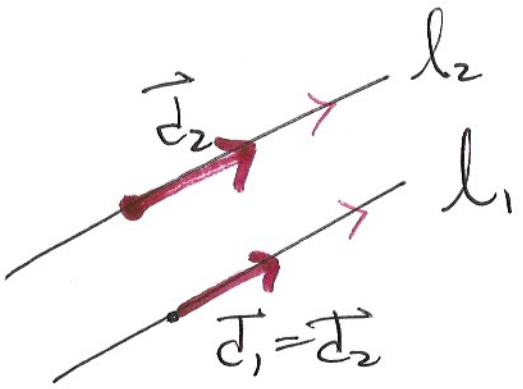
$$\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c} \quad \text{IF } a,b,c \neq 0$$

FIND SYMMETRIC EQUATIONS OF THE LINE THROUGH $(7, 0, -1)$
WITH DIRECTION VECTOR $\vec{d} = \langle -4, -1, 2 \rangle$

$$\frac{x-7}{-4} = \frac{y-0}{-1} = \frac{z-(-1)}{2}$$

$$\frac{7-x}{4} = -y = \frac{z+1}{2}$$

FIND SYMMETRIC EQ'NS OF LINE THROUGH $(0, 5, -6)$ $\leftarrow l$
AND PARALLEL TO $\frac{2x+1}{1} = \frac{y-7}{2} = \frac{4-z}{1}$ $\leftarrow l_2$



LET $\vec{d}_1 = \vec{d}_2 = \langle \frac{1}{2}, 2, -1 \rangle$
OR $2\langle \frac{1}{2}, 2, -1 \rangle$

$\frac{2x+1}{1}$ $\frac{1}{2}$
$\frac{1}{2}$

$$\frac{x+\frac{1}{2}}{\frac{1}{2}} = \frac{y-7}{2} = \frac{z-4}{-1}$$

$\frac{x+\frac{1}{2}}{\frac{1}{2}}$

$$\vec{d}_1 = \langle 1, 4, -2 \rangle$$

$$\frac{x-0}{1} = \frac{y-5}{4} = \frac{z-(-6)}{-2}$$

$$x = \frac{y-5}{4} = \frac{-z-6}{2}$$